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The Sound of M-Theory

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Abstract

We consider sound propagation on M5- and M2-branes in the hydrodynamic limit. In particular, we look at the low energy description of a stack of N M-branes at finite temperature. At low energy, the M-branes are well described, via the AdS/CFT correspondence, in terms of classical solutions to the eleven dimensional supergravity equations of motion. From this gravitational description, we calculate Lorentzian signature two-point functions of the stress-energy tensor on these M-branes in the long-distance, low-frequency limit, i.e. the hydrodynamic limit. The poles in these Green's functions show evidence for sound propagation in the field theory living on the M-branes.

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1 Introduction

The interacting, superconformal field theories living on a stack of N M2- or M5-branes are not well understood. An improved understanding of these M-branes should lead eventually to a better understanding of M-theory itself, a theory that encompasses all the different superstring theories and is one of the best hopes for a quantum theory of gravity. While the full M-brane theories remain mysterious, the low energy, large N behavior is conjectured to be described well, via the AdS/CFT correspondence [1, 2, 3], by certain classical solutions to eleven dimensional supergravity equations of motion. Recent work on AdS/CFT correspondence by Son, Starinets, Policastro, and the author [4, 5, 6, 7] provides a prescription for writing Lorentzian signature correlators for these types of theories. In [8], this prescription was used to calculate viscosities and diffusion constants for M-brane theories in this low energy limit, thus generalizing the work of [5] for D3-branes. This paper completes the program started in [8] by considering sound waves on M2- and M5-branes, and also generalizes similar work by Son, Starinets, and Policastro [6] for D3-branes.

The behavior of any field theory perturbed a small amount away from thermal equilibrium is expected to be described well by fluid mechanics [9]. In particular, in this limit the size of the fluctuations are small compared to the temperature. Moreover, fluid mechanics supports sound waves. Thus, we expect the finite temperature field theory living on a stack of M-branes to conduct sound.

The existence of sound waves has strong implications for the two-point function of the stress-energy tensor. In particular, we expect to see poles in some of the components of this two-point function from which we can extract the speed of sound and a damping term.

Using the techniques described in [4, 7], we calculate the the relevant components of the two-point function of the stress-energy tensor in these M-brane theories using their gravity duals.[1] We find a sound wave pole exactly of the form predicted by fluid mechanics:

There were two main results of [8]. The first was that finite temperature AdS/CFT correspondence can be used to describe M-brane theories in the hydrodynamic limit. Indeed,

[1]For a different perspective on the prescription for writing field theory, Lorentzian signature Green's functions from gravity using AdS/CFT, see [10].

the AdS/CFT description passed some nontrivial consistency checks, among them two independent derivations of the viscosity. The second result was more evidence in support of the Lorentzian signature prescription proposed in [4, 7] for calculating field theory correlators from gravity.

The two main results of this paper are largely the same. With sound waves and more internal consistency checks, among them a third independent calculation of the viscosity, we find additional reason to believe that AdS/CFT describes well the hydrodynamic limit of M-brane theories. This paper also provides further support for the Lorentzian signature prescription of [4, 7].

We begin in section 2 by reviewing shear modes and sound waves in fluid mechanics and the constraints they place on the two-point function of the stress-energy tensor. In section 3, we discuss the field theory Ward identities. We show how these identities constrain the components of the stress-energy two-point function. In particular, the identities essentially constrain the infrared behavior of the Green's functions up to the location of the sound wave pole and a thermal factor.

Section 4 contains the body of the paper, the calculations of the stress-energy two-point functions from gravity. These calculations produce the correct sound wave pole and match precisely the Green's functions obtained from the Ward identities.

2 Sound Waves and the Hydrodynamic Limit

In the hydrodynamic limit, small perturbations of the stress-energy tensor from thermal equilibrium correspond to sound and shear modes. We review the differential equations governing these modes. We are interested in the thermal field theories living on M2- and M5-branes. Thus the stress tensor will be respectively $2+1$ or $5+1$ dimensional.

One of the differential equations is the conservation condition $\partial_\mu T^{\mu\nu} = 0$. The second equation is the linearized form of the stress-energy tensor, which describes how small, slowly varying velocity gradients produce extra stresses in the fluid because of viscosity [9]:

$$T^{ij} = P\delta^{ij} - \frac{\eta}{\epsilon + P}\left(\partial_i T^{0j} + \partial_j T^{0i} - \frac{2}{d}\delta^{ij}\partial_k T^{0k}\right) \tag{1}$$

where $P = \langle T^{ii} \rangle$ is the pressure, $\epsilon = \langle T^{00} \rangle$ is the energy density, and η is the viscosity. The dimension d is 2 for the M2-branes and 5 for the M5-branes, and i, j, and k run only over the spatial indices. To analyze these equations, it is helpful to split T^{0i} into longitudinal and transverse parts (see for example [11]):

For the transverse or shear modes, the conservation condition and the linearized stress-energy tensor combine to give

For the longitudinal or sound modes, one finds that

3 Ward Identities

The one- and two-point functions of the stress-energy tensor are highly constrained by Ward identities. In particular, we will find that the components of the two-point function relevant to sound propagation are constrained up to one free parameter α. In the following, we will review the relevant Ward identities and try to motivate a particular choice of α.

In a flat metric, the Ward identity for $\langle T^{\mu\nu}\rangle$ is conservation of energy, $\partial_\mu \langle T^{\mu\nu}\rangle = 0$. For our thermal field theories, $\langle T^{\mu\nu}\rangle$ is a constant with $\langle T^{00}\rangle = \epsilon$ and $\langle T^{ij}\rangle = P\delta^{ij}$.

For the two-point function,

We now identify the particular components of $G^{\mu\nu\lambda\rho}$ relevant to sound propagation. We choose our sound wave to have wave vector $(\omega, q, 0, 0, \dots)$. In this frame, we are only interested in components of $T^{\mu\nu}$ which are invariant under rotations in the remaining $d-1$ spatial directions perpendicular to the $x_1 = x$ axis, i.e. T^{tt}, T^{tx}, T^{xx}, and $T^{aa} = T^{x^2x^2} + \ldots + T^{x^dx^d}$. Therefore, there are ten independent correlators G^{AB} where A and B are tt, xx, tx, or aa.

There are twelve Ward identities for these correlators, nine of which are linearly independent. Thus, there is a one parameter family of Green's functions G that satisfy these Ward identities. Let G_1 be a particular solution to the identities such that the complex conjugate $G_1^* \neq G_1$. As the identities have real coefficients, G_1^* will also be a solution, and we can write the most general solution as

We now find a particular solution G_1 such that $G_1 \neq G_1^*$ with a desirable quality: G_1 will be closely related to the retarded Green's function G_R. We expect the only singularities of the *retarded* Green's functions, G_R, to be simple poles at $\omega = \pm q/\sqrt{d} - i\Gamma q^2 + \mathcal{O}(q^3)$, corresponding to sound propagation; $\Gamma > 0$ is the damping term. Let us assume that G_1 has the same pole structure as G_R. With this additional constraint, the correlators become

To compare with the gravity calculations to come, we need an expression for the Feynman Green's function G_F. As the G in the Ward identities are defined by taking functional derivatives of an appropriately defined path integral, we expect that one of these solutions to the Ward identities should be identified with the Feynman Green's functions, G_F. There must be some α in (**??**) for which $G = G_F$. From the definitions of the various Green's functions and the Kubo-Martin-Schwinger (KMS) condition, we know that $G_F = (1+n)G_R - nG_R^*$, up to real contact terms, where $n = (\exp(\omega/T) - 1)^{-1}$. Thus it seems likely that $G = G_F$ in (**??**) when $\alpha = n$, i.e.

4 Two-Point Functions from Gravity

In the following, we check that a gravitational calculation produces stress-energy correlators that obey the Ward identities. Indeed, we will find that the M2- and M5-brane correlators calculated from gravity have precisely the form (**??**).

4.1 M2-branes

The background metric is

To study the two-point functions of the stress-energy tensor, we introduce perturbations to this background metric of the form $g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$. We choose a gauge where $h_{u\mu} = 0$. We consider waves that move in the x direction. Such waves break into two categories, depending on whether the metric perturbations are odd or even under $y \to -y$. The odd perturbations correspond to diffusive shear waves and were treated in [8]. The even perturbations correspond to sound waves and will be treated here. For sound waves, the nonzero elements of $h_{\mu\nu}$ are h_{xx}, h_{yy}, h_{tt}, and h_{xt}.

We decompose the metric perturbations into their Fourier components

To first order in the $h_{\mu\nu}$, the Einstein equations are

The system above is not linearly independent, but can be reduced to the following four linearly independent differential equations

These systems of differential equations are invariant under three residual gauge transformations. These pure gauge solutions are linear combinations of H^I, H^{II}, and H^{III} whose explicit forms are

$$H^I_{xt} = -\omega_2 \; , \tag{2aaa}$$

$$H^I_{xx} = 2q_2 \; , \tag{2aab}$$

$$H^{II}_{tt} = 2\omega_2 \; , \tag{2aba}$$

$$H^{II}_{xt} = q_2 f \; , \tag{2abb}$$

$$H^{III}_{tt} = -2\omega_2^2 \int \frac{u}{f^{3/2}} du - 4\left(\frac{2+u^3}{f^{1/2}}\right) \; , \tag{2aca}$$

$$H^{III}_{xt} = -\omega_2 q_2 \left(\int \frac{u}{f^{1/2}} du + f \int \frac{u}{f^{3/2}} du \right) \; , \tag{2acb}$$

$$H^{III}_{xx} = 2q_2^2 \int \frac{u}{f^{1/2}} du - 8f^{1/2} \; , \tag{2acc}$$

$$H^{III}_{yy} = -8f^{1/2} \; . \tag{2acd}$$

There is an incoming solution to these differential equations, where by incoming we mean that the wave fronts at the horizon are purely incoming. The incoming solution to linear order[2] in ω_2 and q_2 is

$$H_{tt}^{inc} = \mathcal{O}(\omega_2^2, \omega_2 q_2, q_2^2) \ , \tag{2ada}$$

$$H_{s}^{inc} = \mathcal{O}(\omega_2^2, \omega_2 q_2, q_2^2) \ , \tag{2adb}$$

$$H_{xt}^{inc} = (1-u)^{-i\omega_2/6} \left(-\frac{iq_2}{6} f(u) + \mathcal{O}(\omega_2^2, \omega_2 q_2, q_2^2) \right) \ , \tag{2adc}$$

$$H_{yy}^{inc} = (1-u)^{-i\omega_2/6} \left(1 - \frac{i\omega_2}{6} \ln \frac{1+u+u^2}{3} + \mathcal{O}(\omega_2^2, \omega_2 q_2, q_2^2) \right) \ . \tag{2add}$$

There is also naturally an outgoing solution at the horizon which is just the complex conjugate of $H^{inc} = (H^{out})^*$. The nonperturbative piece $(1-u)^{\pm i\omega_2/6}$ enforces the incoming or outgoing boundary condition at the horizon $u = 1$, as discussed in more detail in [5, 8].

Setting the boundary conditions is a delicate issue. In [7], it was argued that the correct boundary conditions for calculating field theory propagators using AdS/CFT are purely incoming for positive frequency modes and purely outgoing for negative frequency modes where positive and negative frequency are defined with respect to Kruskal time. Moreover, the full Penrose diagram for this black hole background and a second boundary were used to reproduce the 2×2 matrix of Schwinger-Keldysh propagators. For simplicity, we will focus on deriving just the Feynman propagator, i.e. the first entry in this 2×2 matrix, in what follows.

Let $H(u)$ be the solution to the system of ODE's which contains no H^{out} component, i.e.

Solving this system (**??**) of equations to linear order in ω_2 and q_2, one finds that the constants a, b, c, and d are all proportional to

4.2 M5-branes

The calculations for the M5-brane are very similar. The background metric is

[2] There is some confusion in the literature concerning the order to which H^{inc} must be computed to see the damping of the sound wave. In [6], the authors computed H^{inc} to quadratic order, but we will see that the damping already appears at linear order.

We again decompose the metric perturbations into their Fourier components (**??**). We also introduce the dimensionless energy and momentum

$$\omega_5 = \frac{3\omega}{4\pi T} \quad ; \quad q_5 = \frac{3q}{4\pi T} \ .$$

To first order in the $h_{\mu\nu}$, the Einstein equations are

This system is equivalent to the four linearly independent equations

These systems of differential equations are invariant under three residual gauge transformations. These pure gauge solutions are linear combinations of H^I, H^{II}, and H^{III} whose explicit forms are

$$H^I_{xt} = -\omega_5 \ , \qquad \text{(2aeaaa)}$$

$$H^I_{xx} = 2q_5 \ , \qquad \text{(2aeaab)}$$

$$H^{II}_{tt} = 2\omega_5 \ , \qquad \text{(2aeaba)}$$

$$H^{II}_{xt} = q_5 f \ , \qquad \text{(2aeabb)}$$

$$H^{III}_{tt} = -2\omega_5^2 \int \frac{1}{f^{3/2}} du - \left(\frac{1+2u^3}{f^{1/2}}\right) \ , \qquad \text{(2aeaca)}$$

$$H^{III}_{xt} = -\omega_5 q_5 \left(\int \frac{1}{f^{1/2}} du + f \int \frac{1}{f^{3/2}} du\right) \ , \qquad \text{(2aeacb)}$$

$$H^{III}_{xx} = 2q_5^2 \int \frac{1}{f^{1/2}} du - f^{1/2} \ , \qquad \text{(2aeacc)}$$

$$H^{III}_{yy} = -f^{1/2} \ . \qquad \text{(2aeacd)}$$

The incoming solution to linear order in ω_2 and q_2 is

$$H^{inc}_{tt} = \mathcal{O}(\omega_5^2, \omega_5 q_5, q_5^2) \ , \qquad \text{(2aeada)}$$

$$H^{inc}_{s} = \mathcal{O}(\omega_5^2, \omega_5 q_5, q_5^2) \ , \qquad \text{(2aeadb)}$$

$$H^{inc}_{xt} = (1-u)^{-i\omega_5/3}\left(-\frac{4iq_5}{3} f(u) + \mathcal{O}(\omega_5^2, \omega_5 q_5, q_5^2)\right) \ , \qquad \text{(2aeadc)}$$

$$H^{inc}_{yy} = (1-u)^{-i\omega_5/3}\left(1 - \frac{i\omega_5}{3} \ln \frac{1+u+u^2}{3} + \mathcal{O}(\omega_5^2, \omega_5 q_5, q_5^2)\right) \ . \qquad \text{(2aeadd)}$$

The solution to this system of ODE's with pure incoming boundary conditions at the horizon is thus a linear combination of

4.3 Two-Point Functions from the Boundary Action

Although one can see immediately from the constants a, b, c, and d in the solution to the bulk graviton (**??**) and (**??**) that the two-point functions of the stress-energy tensor will have poles that are consistent with sound wave propagation, it is a worthwhile exercise to work out the two-point functions explicitly using the AdS/CFT prescription. One benefit is that we will be able to check the two-point functions calculated from the gravity against the formulae we derived from the Ward identities (**??**)–(**??**).

In order to use the AdS/CFT prescription, we need to find the boundary action for our gravitational systems. The full action is

Using the equations of motion, the full action can be reduced to a boundary term. This boundary action for the M2-branes is

$$\begin{aligned} S_b &= \frac{P}{8}\int d^3x\,[-8 - 8H_{tt} + 4H_{xx} + 4H_{yy} \\ &\quad -\frac{2}{\epsilon^2}\left(H_{tt}(H_{xx}+H_{yy}) + H_{xx}H_{yy} + H_{xt}^2\right)' \\ &\quad -(H_{xx}+H_{yy})H_{tt} - H_{xx}^2 + 2H_{xx}H_{yy} - H_{yy}^2 + 2H_{tt}^2 - 8H_{xt}^2]\ , \end{aligned} \qquad (2aeae)$$

where the constant $P = 8\sqrt{2}\pi^2 T^3 N^{3/2}/81$ is the pressure.

For the M5-branes, the boundary action is

$$\begin{aligned} S_b &= \frac{P}{8}\int d^6x\big[-8 - 20H_{tt} + 4H_{xx} + 4H_{aa} \\ &\quad -\frac{1}{\epsilon^2}\left(4H_{tt}(H_{xx}+H_{aa}) + \frac{3}{2}H_{aa}^2 + 4H_{aa}H_{xx} + 4H_{xt}^2\right)' \\ &\quad -4H_{tt}(H_{xx}+H_{aa}) - H_{xx}^2 + 2H_{aa}H_{xx} + \frac{1}{2}H_{aa}^2 + 5H_{tt}^2 - 20H_{xt}^2\Big] \end{aligned} \qquad (2aeaf)$$

where $P = 2^6\pi^3 N^3 T^6/3^7$. We have defined $H_{aa} \equiv \sum_{i=2}^{5} H_{x^i x^i} = 4H_{yy}$.

Before looking at the two-point functions, we make two easy and reassuring observations. First, the constant term in the boundary actions is exactly the free energy density, i.e. minus the pressure, multiplied by the volume, as it should be [14]. As an independent check, the pressure can be calculated from the entropy S using the fact that $S = \partial P/\partial T$. The entropy for M2- and M5-branes was first determined in [15].

The second observation concerns the one-point functions. By definition

The two-point functions are defined similarly:

5 Conclusion

We have succeeded in calculating components of the stress-energy tensor two-point functions relevant for sound propagation on M2- and M5-branes in the hydrodynamic limit. These two-point functions have poles in the complex frequency plane in precisely the locations predicted by hydrodynamics. Moreover, the two-point functions calculated from gravity satisfy the field theory conformal Ward identities, providing an elaborate check of the AdS/CFT correspondence at finite temperature. The agreement between gravity and field theory gives further support for the Lorentzian signature prescription of [4, 7] for calculating correlation functions.

Presumably, a general statement can be made that correlation functions calculated from Schwarzschild black holes in AdS of an arbitrary dimension will show evidence of sound propagation. We have checked this statement for M2- and M5-branes. Previously, the statement had been checked only for D3-branes [6].

To see a familiar phenomenon such as sound emerge from a poorly understood theory, such as those that are meant to describe M-branes, is in the author's mind interesting and remarkable. In the M5-brane case, there is not even a Lagrangian description, and yet we can understand sound waves.

An interesting direction to pursue is gauge/gravity duality for non-conformal field theories in the hydrodynamic limit. The diffusion constants and viscosities calculated should exhibit a scale dependence. One candidate gauge/gravity duality where we have high temperature asymptotics is the finite temperature Klebanov-Tseytlin solution [16]. We leave investigation of these issues for future work.

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A The Bulk Graviton and Correlation Functions

We present here in more detail some intermediate steps for calculating stress-energy two-point functions for M-branes.

By assumption, the bulk graviton components H_{xt}, H_{tt}, H_s, and H_{yy} take on the values H^0_{xt}, H^0_{tt}, H^0_s, and H^0_{yy} at the boundary $u = 0$. However, to calculate the two-point functions, we also need an expression for $H(u)'$ at the boundary. Define $P_{\mu\nu} = H_{\mu\nu}Q$ where $Q = 2\omega_2^2 - q_2^2 + \frac{1}{3}iq_2^2\omega_2$. Then, for the M2-branes, from the solution to (**??**), we find

$$
\begin{aligned}
P'_{tt}\big|_{u=\epsilon} = -\,P'_s\big|_{u=\epsilon} &= \epsilon^2\left(3(2\omega_2 q_2 H^0_{xt} - q_2^2 H^0_{tt} + \omega_2^2 H^0_s) + iq_2^2\omega_2 H^0_{yy}\right) + \ldots && \text{(2aeaga)}\\
P'_{yy}\big|_{u=\epsilon} &= \frac{\epsilon^2}{2}\Big(-3(2\omega_2 q_2 H^0_{xt} - q_2^2 H^0_{tt} + \omega_2^2 H^0_s) - 2i\omega_2^2 q_2 H^0_{xt} && \text{(2aeagb)}\\
&\quad +2i\omega_2(\omega_2^2 - q_2^2)H^0_{yy} - i\omega_2^3 H^0_s + iq_2^2\omega_2 H^0_{tt}\Big) + \ldots && \text{(2aeagc)}\\
P'_{xt}\big|_{u=\epsilon} &= \frac{\epsilon^2}{2}\Big(3q_2(2q_2 H^0_{xt} - 2\omega_2 H^0_{tt} + \omega_2 H^0_s) && \text{(2aeagd)}\\
&\quad -2iq_2^2\omega_2 H^0_{xt} - iq_2\omega_2^2 H^0_s + 2iq_2\omega_2^2 H^0_{yy}\Big) + \ldots && \text{(2aeage)}
\end{aligned}
$$

The ellipses denote higher order terms in ϵ, ω_2, and q_2. The leading term in ϵ is precisely of the form to cancel the leading $1/\epsilon^2$ divergence in the boundary action (2aeae).

For the M5-branes, there is an analogous expression for $H(u)'$. Defining $P_{\mu\nu} \equiv H_{\mu\nu}Q$, where now $Q = 5\omega_5^2 - q_5^2 + \frac{8}{3}iq_5^2\omega_5$, one finds from (**??**) that

$$P'_{tt}|_{u=\epsilon} = -\left.P'_s\right|_{u=\epsilon} = \epsilon^2\left(\frac{15}{2}(2\omega_5 q_5 H^0_{xt} - q_5^2 H^0_{tt} + \omega_5^2 H^0_s) + 20iq_5^2\omega_5 H^0_{yy}\right) + \ldots \tag{2aeaha}$$

$$\left.P'_{yy}\right|_{u=\epsilon} = \epsilon^2\Big(-\frac{3}{2}(2\omega_5 q_5 H^0_{xt} - q_5^2 H^0_{tt} + \omega_5^2 H^0_s) + 5i\omega_5(\omega_5^2 - q_5^2)H^0_{yy} \tag{2aeahb}$$

$$-i\omega_5^3 H^0_s + iq_5^2\omega_5 H^0_{tt} - 2i\omega_5^2 q_5 H^0_{xt}\Big) + \ldots \tag{2aeahc}$$

$$\left.P'_{xt}\right|_{u=\epsilon} = \epsilon^2\Big(\frac{3}{2}q_5(2q_5 H^0_{xt} - 5\omega_5 H^0_{tt} + \omega_5 H^0_s) \tag{2aeahd}$$

$$-8iq_5^2\omega_5 H^0_{xt} - 4iq_5\omega_5^2 H^0_s + 20iq_5\omega_5^2 H^0_{yy}\Big) + \ldots \tag{2aeahe}$$

The next step in calculating the stress-energy two-point functions is to insert these expressions for $H(u)'$ into the boundary action (2aeae) or (2aeaf). The terms in the boundary action that produce the two-point functions can be represented schematically as H^2 and HH'. The H^2 pieces produce constant contact terms. The HH' terms, on the other hand, are responsible for the sound wave pole structure.

There is an apparent ambiguity how to proceed. Looking more closely, it should not matter at the level of the action whether the HH' are written as $H(-k,u)H(k,u)'$ or $H(k,u)H(-k,u)'$. However, sending $k \to -k$ sends our solution $H(u) \to H(u)^*$.

Recall that according to [7], we should be substituting $H_K(u) = (1+n)H(u) - nH(u)^*$ into the boundary action, and not just $H(u)$. $H_K(u)$ is invariant under the transformation $k \to -k$. Rather than carry these cumbersome thermal factors n along, we will adopt the equivalent procedure of always writing HH' as $H(-k,u)H(k,u)'$ and using the simpler incoming solution $H(u)$. Let S_b be the resulting boundary action. We can reconstruct G_F by taking functional derivatives of $(1+n)S_b - nS_b^*$ instead of just S_b.

In this spirit, for the M2-branes, the HH' pieces of the boundary action become

$$\begin{aligned}\left.S_b\right|_{HH'} = \; & \frac{P}{8}\left(2\omega_2^2 - q_2^2 + \frac{1}{3}iq_2^2\omega_2\right)^{-1}\Big(12\omega_2 q_2\left(2H^0_{xt}H^0_{tt} - H^0_{xt}H^0_{xx} - H^0_{xt}H^0_{yy}\right)\\
& +3\omega_2^2\left(2H^0_{tt}H^0_{xx} + 2H^0_{tt}H^0_{yy} - (H^0_{xx})^2 - (H^0_{yy})^2 - 2H^0_{xx}H^0_{yy}\right)\\
& +3q_2^2\left(H^0_{tt}H^0_{xx} + H^0_{tt}H^0_{yy} - 4(H^0_{xt})^2 - 2(H^0_{tt})^2\right)\\
& +iq_2^2\omega_2\left(3H^0_{yy}H^0_{tt} - 2(H^0_{yy})^2 + 4(H^0_{xt})^2 - H^0_{tt}H^0_{xx}\right)\\
& +4i\omega_2^2 q_2 H^0_{xt}\left(H^0_{xx} - H^0_{yy}\right) + i\omega_2^3\left(H^0_{xx} - H^0_{yy}\right)^2\Big)\,.\end{aligned} \tag{2aeai}$$

The quadratic terms in H are tacitly assumed to be of the form $H(-\omega,-q)H(\omega,q)$. Also, the integral over ω and q has been suppressed.

For the M5-branes, the HH' piece of the boundary action is similarly

$$\begin{aligned} S_b|_{HH'} &= \frac{P}{4}\left(5\omega_5^2 - q_5^2 + \frac{8}{3}iq_5^2\omega_2\right)^{-1}\Big(12\omega_5 q_5\left(5H^0_{xt}H^0_{tt} - H^0_{xt}H^0_{xx} - 4H^0_{xt}H^0_{yy}\right) \\ & +3\omega_5^2\left(5H^0_{tt}H^0_{xx} + 20H^0_{tt}H^0_{yy} - (H^0_{xx})^2 - 16(H^0_{yy})^2 - 8H^0_{xx}H^0_{yy}\right) \\ & +3q_5^2\left(H^0_{tt}H^0_{xx} + 4H^0_{tt}H^0_{yy} - 4(H^0_{xt})^2 - 5(H^0_{tt})^2\right) \\ & +8iq_5^2\omega_2\left(6H^0_{yy}H^0_{tt} - 5(H^0_{yy})^2 + 4(H^0_{xt})^2 - H^0_{tt}H^0_{xx}\right) \\ & +32i\omega_5^2 q_5 H^0_{xt}\left(H^0_{xx} - H^0_{yy}\right) + 8i\omega_5^3\left(H^0_{xx} - H^0_{yy}\right)^2\Big) \ . \end{aligned} \tag{2aeaj}$$

Recall that $H_{yy} = H_{x^2x^2} = H_{x^3x^3} = H_{x^4x^4} = H_{x^5x^5}$ for these M5-branes.

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